Influence of Characteristics of Crystals on Performance Of Tracking Loops

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***Abstract*—A significant challenge has been existed concerning the relationship between the manufacturing costs and the high performance of the handheld GNSS terminals. The crystals of the oscillators represent one the most expensive component in the terminals even, the crystals used in them, can often be called rubbish ones. The work is pending to develop a method including a mathematical / software procedure to estimate the expected performance of the tracking loops (or other closed loop systems having crystal oscillators) in the terminals. The future creates more new challenges since e.g. the indoor - and the poor conditions –navigation will have the more and more important role because, for example, the number and transmitted power of the satellite signals are expected to increase. The theoretical studies of Nokia Technology Platform and MIKES have concentrated on the knowledge available in the existing literature to build up a model comprising of the most essential noise processes and characterizing parameters of tracking loop(s), as noise bandwidth, gain(s), attenuation, etc. The tool will include mathematical algorithms and the final target is to compute for desired tracking loops an optimized solution subject to certain required restriction(s), term(s) or / and conditions. In the case of the simplest solutions, the analytical ones can be derived and the functionality of the tool can be verified by these cases. Experimental computations have been carried out to study the functionality in the more complicated cases and the output of the tool has been observed to be a relevant one. The procedure can be carried out by different spread sheet programs or e.g. by Matlab.**

I. INTRODUCTION

GNSS (Global Navigation Satellite System) receivers have already been developed tens of years. Still, one essential problem is the instability of main / local clock maintaining the timing of different units of receiver as the carrier and code tracking loops. To find out and to estimate the influence of used crystal oscillator(s) on the performance of tracking loops, a mathematical tool is pending to be created. The tool exploits the power series model of the noise process and the system theoretic features of tracking loops to generate term by term the error variances (powers) of the noise components.

A tracking loop can be presented in the control and system theoretical sense as follows:

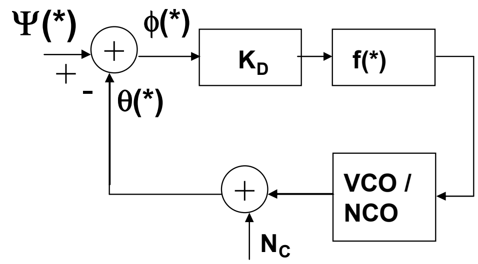

Fig1. System theoretic model of a tracking loop

where,

Ψ is sky signal
θ is replica of true signal
Φ is error signal
K_D is discriminator
f is loop filter
NCO is Numerically Controlled Oscillator
N_C is noise process from clock crystal
VCO is Voltage Controlled Oscillator
* is t in analogue systems and kT in discrete ones
where
t is continued time
T is sampling time
k is integer, $-\infty < k < \infty$

The fundamental problem is to find out a suitable measure to estimate the performance of tracking loops when the changes of the dynamics of clock crystals effect on them. A proper and suitable candidate is the powers (variances) of the frequency components of a relevant noise process, especially, when the needed mathematical methods, models and tools are available.

II. THEORY AND MATHEMATICS

Let consider the system in Fig1, where the N_c describes a relevant noise process generated by the clocking crystal operation. The target is to solve the influence of this noise process on the error signal Φ.

Let exploit a general system to derive a mathematical procedure to the noise power of N_c.

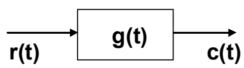

Fig2. Generic basic system

The output and input of the basic system are related to each others for example by Equation

$$c(t) = \int_{-\infty}^{\infty} r(t-\tau)g(\tau)d\tau \tag{1}$$

The relative cross-correlation function of the input and output is achieved by

$$\phi_{rc}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} r(t)c(t+\tau)dt \tag{2}$$

Substituting for c(t) from Eq.1 and solving achieves

$$\phi_{rc}(\tau) = \int_{-\infty}^{\infty} \phi_{rr}(\tau-\alpha)g(\alpha)d\alpha \tag{3}$$

The Fourier transform of the autocorrelation gives the power density spectrum, thus

$$\Phi_{rc}(j\omega) = G(j\omega)\Phi_{rr}(j\omega) \tag{4}$$

in which, the G(jω) is called transfer function of the system.

On the other hand,

$$\phi_{cc}(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} c(t)c(t+\tau)dt \tag{5}$$

By substituting the Eq. 1 into Eq. 5 it is achieved

$$\phi_{cc}(\tau) = \int_{-\infty}^{\infty} \phi_{rr}(\tau+\alpha-\beta)g(\alpha)g(\beta)d\alpha d\beta \tag{6}$$

This yields the relationship of the power spectra of the input and output as follows

$$\Phi_{cc}(j\omega) = G(j\omega)G(-j\omega)\Phi_{rr}(j\omega) \tag{7}$$

Or

$$\Phi_{cc}(j\omega) = |G(j\omega)|^2 \Phi_{rr}(j\omega) \tag{8}$$

$$c^2(t)\frac{1}{2\pi} = \int_{1-\infty}^{\infty} |G(j\omega)|^2 \Phi_{rr}(j\omega)d\omega \tag{9}$$

Let now consider a system with a feedback, as follows

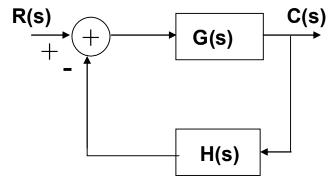

Fig3. Generic system with feedback

In practice, nothing else except the transfer function of the system has changed and the basic idea is the same and now, the form of the power density spectrum of the output is

$$\Phi_{cc}(j\omega) = \left|\frac{G(j\omega)}{1+G(j\omega)H(j\omega)}\right|^2 \Phi_{rr}(j\omega) \tag{10}$$

The signal power of the output c(t) is achieved as in the former case of the forward system.

Let next consider the system described in Fig1. Let the variables and components considered to be transformed (Laplace-, Fourier- or z –one) then the system and control theoretic description for the error signal of the system is

$$\phi(*) = \frac{1}{1+K_D f(*)k_0 U(*)}\Psi(*) - \frac{1}{1+K_D f(*)k_0 U(*)}N_c(*) \tag{11}$$

where

K_D is G(jω)

f k_0U is H(jω)

and the asterisk (*) refers to the variable of the used transform (s, jω or z) and the variable U is unit step.

Now, the term

$$\frac{1}{1+K_D f(*)U(*)}N_c(*) = E(*)N_c(*) \tag{12}$$

presents the influence of the clock on the system with the noise input of Nc (to the system) and by the transfer function $1/(1+K_D f(*)U(*))$ and the respective phase error power of the output signal is

$$\sigma_\phi^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} |E(j\omega)|^2 S_\Psi(j\omega)d\omega \tag{13}$$

And,

$$\sigma_\phi^2 = \frac{1}{2\pi}\oint E(z)E(\frac{1}{z})S_\Psi(z)\frac{1}{z}dz \qquad (14)$$

in the analogue and discrete systems, respectively. The variable of S is the power density spectrum of Nc. In practice and, especially, in the numerical solution, the integral of the z –transformed expressions is easier to solve by writing it in the form where from the additive terms can be recognized the respective Fourier –transforms and then replacing the z –transformed terms with the Fourier –transformed ones and then process the path integral by Eq. 13. The solution is the envelope curve of the solution of Eq. 14.

But, these equations are including the power density spectra of the noise process and the next task is to find a relevant expression for these quantities.

It is known that the power density spectra of the phase and frequency noise processes are bounded to each others by the Eq.

$$S_\Psi(\omega) = \frac{1}{\omega^2} S_{\dot{\Psi}}(\omega) \qquad (15)$$

A well-known noise model with the components of different frequencies can be expressed as follows:

$$S_\Psi(\omega) = \frac{1}{\omega^2}\sum_{i=-\infty}^{\infty} h_i|\omega|^i \qquad (16)$$

which can be limited to the most essential components of the phase noise process by

$$S_\Psi(\omega) = \frac{1}{\omega^2}\sum_{i=-2}^{2} h_i|\omega|^i \qquad (17)$$

including the components of phase white noise, phase flickering, phase random walk or frequency white noise, frequency flickering and frequency random walk with the values of i = 2, 1, 0, -1, -2, respectively.

Two different systems (tracking loops) were deployed, an analogue and a digital one as follows:

analogue one:

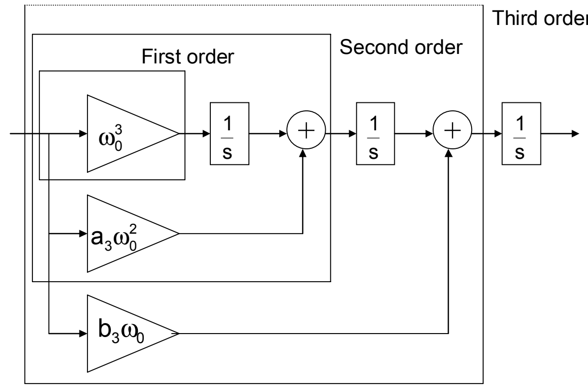

Fig4. Analogue closed loop system where the subsystems of the different orders are framed.

and, digital one:

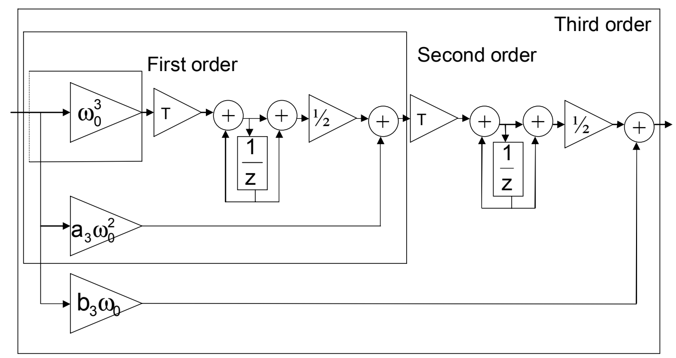

Fig5. Digital closed loop system where the subsystems of the different orders are framed.

III. Practical Procedure to Solve Error Power

As the preceding description detected the essential features effecting on the noise performance of closed tracking loop systems, are the loop filter with the gain of the discriminator and the VCO / NCO and the noise process.

Thus, the characteristics of the loop filter should be studied to scale the changes in the output values of closed loops due to changes in the values of the parameters of the loop filters. A good way is to compare the results achieved by different kind of optimizing procedures based on noise bandwidth, Wiener filter concept etc with the results of typical filters. The loop filters are a function of polynomials in the numerator and in the denominator and, the order of the polynomial in the denominator being as high or higher than that of the numerator.

The studied loop filters are as follows:

first order:

$$f(j\omega) = \omega_L \qquad (18)$$

second order:

$$\mathbf{f(j\omega) = \frac{1}{j\omega}\omega_L^2 + a_2\omega_L} \tag{19}$$

third order:

$$\mathbf{f(j\omega) = -\frac{1}{\omega^2}\omega_L^3 + \frac{1}{j\omega}a_3\omega_L^2 + b_3\omega_L} \tag{20}$$

Since the transfer function of f(*) are equivalent (z-transforms converted to the Fourier ones), thus in the discrete case:

first order:

$$\mathbf{f(j\omega) = \omega_L} \tag{21}$$

second order:

$$\mathbf{f(j\omega) = \frac{\omega_L^2 T}{j\omega} + 2a_2\omega_L - \omega_L^2 T} \tag{22}$$

third order:

$$\mathbf{f(j\omega) = \frac{\omega_L^3 T^2}{(j\omega)^2} + \frac{1}{2}\frac{a_3\omega_L^2 T}{j\omega} + \frac{1}{4}(\omega_L^3 T^2 + 2a_3\omega_L^2 T + 4b_3\omega_L)} \tag{23}$$

where

ω_L is the natural angular velocity of loop filter

T is the integration time

The ideal VCO / NCO is a step function but especially, in the analogue case, these components can be imperfect ones with the transfer function of

ideal ones

$$\mathbf{G(\omega)_{VCO} = \frac{k_0}{j\omega}} \tag{24}$$

$$\mathbf{G(z)_{NCO} = k_0\frac{z}{z-1}} \tag{25}$$

an imperfect one, for example,

$$\mathbf{G(\omega)_{VCO} = \frac{k_0}{j\omega + \frac{1}{T_1}}} \tag{26}$$

However, the VCO / NCO increases the order of the denominator the closed tracking loop transfer function by one and thus, the order always is one or higher.

When the transfer functions of the known components are substituted into Eq. 13, the next Eq:s for the phase error powers of the first three orders of tracking loops can be solved:

first order:

$$\mathbf{\sigma_{\phi,i}^2 = \frac{1}{\pi}\int_0^\infty \frac{1}{\omega^2 + A}\frac{h_{-i}\omega_0^2}{|\omega|^{-i}}d\omega} \tag{27}$$

second order:

$$\mathbf{\sigma_{\phi,i}^2 = \frac{1}{\pi}\int_0^\infty \frac{\omega^2}{(\omega_n^2 - \omega^2)^2 + 4\zeta^2\omega_n^2\omega^2}\frac{h_{-i}\omega_0^2}{|\omega|^{-i}}d\omega} \tag{28}$$

third order:

$$\mathbf{\sigma_{\phi,i}^2 = \frac{1}{\pi}\int_0^\infty \frac{\omega^4}{(C\omega - \omega^3)^2 + (D - B\omega^2)^2}\frac{h_{-i}\omega_0^2}{|\omega|^{-i}}d\omega} \tag{29}$$

where

A, B, C and D are the constants calculated from the features of components of respective tracking loop

ζ is the attenuation of the system (tracking loop)

ω_n is natural angular velocity of the system

i is the type of noise process

One large trouble is to find the essential h_i:s for the existing crystals, especially, the i -values 2 and 1.

The interval between the frequency values which are used in the integration is a problem, too, since the high changes can emerge in a rather narrow band and the total band is often very wide as from $-\infty$ to $+\infty$. In the calculations of this paper, it is applied the formulae of

$$\mathbf{f_{n+1} = 1.2 \cdot f_n \quad f_0 = 0.01\,Hz} \tag{30}$$

Now, the error powers for different kind of systems / tracking loops can be calculated by spread sheet programs (as LOTUS, Excel) or by the MATLAB procedures.

IV. TEST RESULTS OF TYPICAL CRYSTAL

The presented results are those of a typical crystal with the nominal frequency of 19.2 MHz. In the third order case, a couple of optimizing procedure is applied to the loop filters (based on the noise bandwidth maximizing and on the Wiener filter concept).

In the table below, "Noise" refers to the certain noise process term; i = 2, 1, 0, -1 and -2, respectively.

Table 1. Analogue system order of 1 with a_2=0, a_3=0, b_3=0 and ω_L=4.

Noise, i	2	1	0	-1	-2
h_i 10^{-22}	191.3	7.3	1.0	1.0	10.5
$\sigma_{\phi,i}^2$ 10^{-9}	>>	52965	83	123	279534

Table 2. Equivalent discrete system order of 1 with a_2=0, a_3=1.1, b_3 =2.4 and ω_L=4.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	52965	83	123	279534

Table 3. Analogue system order of 2 with a_2=1.414, a_3=0, b_3=0 and ω_L=1.88679.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	54133	124	46	368

Table 4. Equivalent discrete system order of 2 with a_2=1.414, a_3=0, b_3=0 and ω_L=1.27470.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	52517	62	62	82631

Table 5. Analogue system order of 3 with a_2=0, a_3 =1.1, b_3=2.4 and ω_L=1.27470.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	54215	174	103	1031

Table 6. Equivalent discrete system order of 3 with a_2=0, a_3=1.1, b_3=2.4 and ω_L=1.88679.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	52639	58	16	75

If the third other loop is optimised for example, to the subject that the noise bandwidth of the loop has the maximum wideness but the system is still stable then analytically, the bandwidth is 18.2 Hz and the results as follows:

Table 7. Analogue system order of 3 with a_2=0, a_3 =1, b_3=2 and ω_L=1.27470.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	55118	260	177	1686

Table 8. Equivalent discrete system order of 3 with a_2=0, a_3=1, b_3=2 and ω_L=1.88679.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	52892	67	17	103

By the Wiener –optimised loop filter is achieved

Table 9. Analogue system order of 3 with a_2=0, a_3 =2, b_3=2 and ω_L=1.27470.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	54742	173	78	563

Table 10. Equivalent discrete system order of 3 with a_2=0, a_3=2, b_3=2 and ω_L=1.88679.

Noise, i	2	1	0	-1	-2
$h_i\ 10^{-22}$	191.3	7.3	1.0	1.0	10.5
$\sigma^2_{\phi,i}\ 10^{-9}$	>>	52892	67	17	103

V. CONCLUSIONS

The phase error power of the difference of the system input and (e.g. VCO / NCO) output estimate is a proper measure and an effective way to approximate the influence of a crystal oscillator on the performance of closed loop system (as e.g. tracking loop) noise behavior. Actually, the procedure itself is tolerable simple and the computing burden is low at the processing power levels of current ordinary PC - computers. Moreover, suitable mathematical tools are available as the spread sheet programs or MATLAB etc.

The output of the procedure is a single unambiguous value but, however, noise processes are employed and the features of the closed loop systems are included into the algorithm. Moreover, the metric measure of the performance is power which is an additive quantity which makes it possible easily to study the influence of noise term by term, separately.

The results driven in the studies revealed an interesting detail since all loops seem to have troubles to attenuate the noise type of -2 –component or the frequency random walk one, including the optimized third order filters.

The values of the results for the noise process component of the type of i = 2 are large since in the preliminary studies it was not reasonable to concentrate on the band limited cases.

In summary, although the studies are preliminary ones the results are encouraging to continue the study.

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